Physical Model-Free Observer for Nuclear Reactors using Differential Neural Networks with Sliding Mode Learning

J. Humberto Pérez-Cruz and Alexander Poznyak
Department of Automatic Control, CINVESTAV-IPN, AP. 14740, Av. Instituto Politécnico Nacional No. 2508, C.P. 07360, México D.F., México. Tel.: +(525) 55 5061 37 41,Fax: +(525) 55 5061 39 82.e-mail: *jperez,apoznyak@ctrl.cinvestav.mx*
(Paper received on August 09, 2006, accepted on September 25, 2006)

Abstract. This paper investigates the problem of uncertain nonlinear state observation in a nuclear reactor when only the input and the output are available. The use of a differential neural network observer with sliding mode learning law is suggested. A very simplified model of the system is initially employed for off-line training of the neural network. When this process of training has finished, the observer can dispense with mathematical model completely and can realize the on-line state estimation within small margin of error despite uncertainty and noise. The efficiency of this technique is illustrated by simulation.

1. Introduction

Nuclear reactors are inherently nonlinear and very complex systems with time-varying parameters depending on a level power, fuel burnup, Xenon isotope production, among others factors. When, in this kind of systems, state observation is required, it is mostly used for two purposes: feedback control and fault detection. In these tasks, the observer employed has often been linear [1], [2]; consequently, its performance is only satisfactory in a small neighborhood of the operation point of the reactor. Nevertheless, if large variations of the system variables are presented, the previous option is not effective any more and some nonlinear state observer is required. In real situations, state estimation turns out to be a non trivial problem because, besides nonlinearity, variation of parameters, uncertainty and inclusive measurement noise must be considered. Moreover, even though parameters were constant, peculiarity of nuclear reactor model prevents its transformation into the, so-called, companion form; hence, common robust techniques such as high gain observers [3] can not be applied. Nonetheless, under these conditions, it is still possible to obtain an acceptable observation using others robust techniques among which it is worth mentioning neural networks (NN) and sliding modes (SM).

NN are an approach that has generated great enthusiasm as a consequence of their capability of functioning adequately despite a partially (or inclusive totally) incomplete information about plant model. NN could be classified as static (feedforward) or as differential (recurrent) ones [4]. In the first kind of networks, a system dynamics is approximated by a static mapping; therefore, the network outputs are uniquely determined by the current inputs and the weights. In contrast, differential neural networks (DNN) incorporate feedback in their structure. So, they overcome many problems associated with first ones such as global extrema search. Furthermore, DNN

Special Issue in Neural Networks and Associative Memories
Research in Computing Science 21, 2006, pp. 181-193

have better properties of approximation.

In recent years, it has been proposed to use neural networks in nuclear reactors particularly for control [5-9] but generally the networks employed have been static or else, in many cases, they have lacked a rigorous proof of stability.

On the other hand, during the last two decades, SM have emerged as other powerful tool for control, identification, and estimation in uncertain environments. Basically, SM consist of the application of a discontinuous control action for reaching and maintaining the dynamics of a system on the, so-called, sliding surface. The major advantages of SM are: low sensitivity to plant parameter variations and disturbance, fast transient behavior, and exponential convergence [10]. However, despite these advantages, only a few applications of this technique have been reported in nuclear literature. So, in [11] a SM observer is used for estimating the external reactivity and the xenon concentration. In [12] is discussed a feedback controller based on SM observer for a space nuclear reactor.

In spite of fruitful research in DNN and SM, very few authors have considered the possibility of combining the advantages from these two techniques for obtaining an observer with better performance [13] and none, to our knowledge, has considered apply this "sliding neuro observer" to field of nuclear processes. Thereby, in this paper, it is suggested to estimate nuclear reactor states using a special kind of DNN which incorporate a switching correction term in their structure and also a sliding mode learning law. Since only the input and the output of reactor are available, a simplified, but imprecise, mathematical model of system is utilized for off-line initial training of DNN. When this process of training has finished, the observer can work without any mathematical model and can realize the on-line state estimation within small margin of error despite uncertainty and noise. The workability of suggested approach is illustrated by a simulation example.

2. Mathematical Model

In general, the nuclear reactor dynamics is described by the following so-called point kinetics equations with six delayed neutron precursor groups [14]:

$$\dot{n}_t = \frac{\rho_t - \beta}{\Lambda} n_t + \sum_{i=1}^{6} \lambda_i C_{i,t} \tag{1}$$

$$\dot{C}_{i,t} = \frac{\beta_i}{\Lambda} n_t - \lambda_i C_{i,t}, \quad i = 1,\dots,6 \tag{2}$$

where n_t is the neutron power (W), $C_{i,t}$ is the power of the ith group delayed neutron precursor (W), ρ_t is the total reactivity, Λ is the effective prompt neutron lifetime (s), λ_i is the radioactive decay constant of ith group neutron precursor (s^{-1}), β_i is the fraction of ith group delayed neutrons, and β is the total delayed neutron fraction ($\beta = \sum_{i=1}^{6} \beta_i$). It is important to mention that the six group point kinetics equations (1) and (2) are in reality a set of seven ordinary differential equations; accordingly, their manipulation can result difficult. However, it is possible to reduce the system order by combining the six precursor groups into an equivalent single group. First, let us define the effective precursor radioactive decay constant λ as

$$\lambda = \frac{1}{\beta}\sum_{i=1}^{6}\beta_i\lambda_i \tag{3}$$

Next, using (3), the equations (1) and (2) can be simplified into a second order system given by

$$\dot{n}_t = \frac{\rho_t - \beta}{\Lambda} n_t + \lambda C_t \tag{4}$$

$$\dot{C}_t = \frac{\beta}{\Lambda} n_t - \lambda C_t \tag{5}$$

where C_t is the equivalent power of all delayed neutron precursors.

Now then, the total reactivity has two components, the external reactivity $\rho_{ext,t}$ and the internal reactivity $\rho_{int,t}$, that is,

$$\rho_t = \rho_{ext,t} + \rho_{int,t} \tag{6}$$

The external reactivity is related to the position of the control bars. Thus, the external reactivity is considered as the control input of the system. The relationship between the external reactivity and the bar position can be represented through an empirical static function. On the other hand, the internal reactivity is associated with the effects of the temperature feedback. These effects can be described [14] by

$$\dot{\rho}_{int,t} = -\alpha K n_t + \alpha K n_0 - \gamma \rho_{int,t} \tag{7}$$

where α the is negative temperature reactivity coefficient (°C^{-1}), K is the reciprocal of the reactor heat capacity (°C/($W{\cdot}s$)), γ is the reciprocal of mean time for heat transfer to the coolant (s^{-1}), and n_0 is the initial power when the external reactivity is equal to zero. For suitability, we consider in this work that $n_0 = 1W$. The equations (4), (5), and (7) constitute a very simplified third order mathematical model of a nuclear reactor. The nominal parameters corresponding to a TRIGA MARK III research reactor located at National Institute of Nuclear Research of México [15] are as follows: $\alpha = 0.01359875\,°C^{-1}$, $\beta = 6.433x10^{-3}$, $\lambda = 0.4024s^{-1}$, $\Lambda = 38\mu s$, $\gamma = 0.2s^{-1}$, $K = 1/5.21045x10^{4}\,°C/(W{\cdot}s)$ whereas the ranges for the variables of the same reactor operating on standard conditions are: n_t from $1W$ to $1.1MW$, C_t from $420.72W$ to $462.79MW$, $\rho_{int,t}$ from -1.4354 to 0, $\rho_{ext,t}$ from 0 to 1.4354. Defining the state coordinates and the control input as $x_{1,t} := n_t$, $x_{2,t} := C_t$, $x_{3,t} := \rho_{int,t}$, $u_t := \rho_{ext,t}$, the equations (4), (5), and (7) can be represented in the standard state variable form

$$\begin{aligned}\dot{x}_{1,t} &= -\frac{\beta}{\Lambda}x_{1,t} + \lambda x_{2,t} + \frac{1}{\Lambda}x_{1,t}x_{3,t} + \frac{1}{\Lambda}x_{1,t}u_t \\ \dot{x}_{2,t} &= \frac{\beta}{\Lambda}x_{1,t} - \lambda x_{2,t} \\ \dot{x}_{3,t} &= -\alpha K x_{1,t} + \alpha K n_0 - \gamma x_{3,t}\end{aligned} \tag{8}$$

3. Observability Analysis

Consider the nonlinear nominal system described by (for the sake of briefness, system (9) is SISO but results presented in this work can be easily extended to MIMO systems)

$$\dot{x}_t = f(x_t, u_t, t),\ y_t = Cx_t \tag{9}$$

where $x_t \in \Re^n$ is the system state at time $t \geq 0$, $y_t \in \Re$ is the system output, $u_t \in \Re$ is the control action, $C \in \Re^{1 \times n}$ is an output matrix and $f : \Re^n \times \Re \times \Re^+ \to \Re^n$.

Definition 1. *The system (9) is said to be observable within the interval time* $[t_0, t_1]$, $t_1 > t_0$ *when the output data* $y(t)$ *determine the initial state* $x(t_0)$ *completely.*

Likewise, let us define the extended output vector as

$$Y_t := \begin{bmatrix} y_t & \dot{y}_t & \cdots & \overset{(n-1)}{y_t} \end{bmatrix}^T \tag{10}$$

and the observability matrix

$$Q = \frac{\partial Y_t}{\partial x_t} \tag{11}$$

Results on observability of system (9) are presented in [16] and references there. These results can be summarized by the following:

Corollary 1. *The system (9) is* ***locally observable*** *in a neighborhood of the point* x_t *at time* t, *if*

$$\det Q \neq 0 \tag{12}$$

We will apply corollary 1 to (8) before trying to realize some observer design for the nuclear reactor. Since in this work it is considered that the only measurable state of (8) is $x_{1,t}$, then the system output is defined as $y_t := x_{1,t}$. Or else, in terms of the state vector, $y_t = \mathbf{C}\mathbf{x}_t$ where $\mathbf{C} := [1\ 0\ 0]$ and $\mathbf{x}_t \in \Re^3$. Next, let us calculate the corresponding extended output vector and observability matrix for (8)

$$Y_t = [y_t \quad \dot{y}_t \quad \ddot{y}_t]^T = [x_{1,t} \quad \dot{x}_{1,t} \quad \ddot{x}_{1,t}]^T$$

$$Q = \frac{\partial Y_t}{\partial x_t} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{\beta}{\Lambda} + \frac{1}{\Lambda} x_{3,t} + \frac{1}{\Lambda} u_t & \lambda & \frac{1}{\Lambda} x_{1,t} \\ \frac{\partial \ddot{x}_{1,t}}{\partial x_{1,t}} & \frac{\partial \ddot{x}_{1,t}}{\partial x_{2,t}} & \frac{\partial \ddot{x}_{1,t}}{\partial x_{3,t}} \end{bmatrix}$$

In accordance with corollary 1, one can deduce that the nominal system (8) is not observable over the manifold

$$\det Q = \left(\frac{\lambda^2}{\Lambda} - \frac{\lambda\beta}{\Lambda^2} - \frac{\gamma\lambda}{\Lambda}\right)x_{1,t} + \frac{\lambda}{\Lambda^2}x_{1,t}x_{3,t} + \frac{\lambda}{\Lambda^2}x_{1,t}u_t + \frac{\lambda^2}{\Lambda}x_{2,t} = 0 \qquad (13)$$

Remark 1. Notice that the manifold (13) is a two-dimensional surface in a 3-dimensional space and, hence, practically any dynamic trajectory (if the control is not a specially oriented for maintaining the dynamics within this manifold) will cross this manifold loosing the observability practically almost everywhere.

4. Differential Neural Network

4.1. Basic assumptions

The uncertain nonlinear system, which states will be reconstructed, is given by

$$\dot{x}_t = f(x_t, u_t, t) + \xi_{1,t},\ y_t = Cx_t + \xi_{2,t} \qquad (14)$$

where x_t, y_t, u_t, f, and C are as in (9) but now the vectors $\xi_{1,t}$ and $\xi_{2,t}$ characterize mixed uncertainties that may include both unmodelled dynamics and deterministic disturbances. Notice that an alternative representation for (14) always could be

$$\dot{x}_t = A^{(0)}x_t + W^{(0)}\sigma(x_t) + B^{(0)}u_t + \tilde{f}_t \qquad (15)$$

where the parameters $A^{(0)} \in \Re^{n\times n}$, $W^{(0)} \in \Re^{n\times n}$, $B^{(0)} \in \Re^{n\times 1}$ are subjected to adjustment, the activation vector-function $\sigma(\cdot) := [\sigma_1(\cdot), ..., \sigma_n(\cdot)]^T$ has sigmoidal components

$$\sigma_j(x) := a_{\sigma j}\left[1 + b_{\sigma j}\exp\left(-\sum_{j=1}^{n} c_{\sigma j}x_{j,t}\right)\right]^{-1} \text{ for } j = 1,\dots,n \qquad (16)$$

and $\tilde{f}_t := f(x_t, u_t, t) - A^{(0)}x_t - W^{(0)}\sigma(x_t) - B^{(0)}u_t + \xi_{1,t}$.

Hereafter it is supposed that the system (14), aside from observability condition given in corollary 1, comply with the following assumptions:

A) System (14) satisfies the (uniform on t) Lipschitz condition, that is,

$$\|f(x,u,t) - f(z,v,t)\| \le L_1\|x - z\| + L_2\|u - v\|$$
$$x, z \in \Re^n;\ u, v \in \Re^m;\ 0 \le L_1, L_2 < \infty$$

B) Admissible controls are bounded, $U^{adm} := \left\{u : \|u\|_\Lambda^2 := u^T\Lambda u \le v_0 < \infty\right\}$, $\Lambda > 0$.

Besides, u_t is such that does not violate the existence of the solution to ODE (14).

C) There exists a control function $\mathring{u}_t$ belonging to the admissible control action set U^{adm} defined in assumption B, such that the uncertain nonlinear system (14) is quadratically stable, namely, there exists a Lyapunov function $V(x)$ such that $\frac{\partial V}{\partial x}\dot{x}_t \leq -n_1\|x_t\|^2, \quad n_1 > 0$ whenever $u_t = \mathring{u}_t$.

D) The mixed uncertainties $\xi_{1,t}$ and $\xi_{2,t}$ are bounded, i.e., $\|\xi_{j,t}\|^2_{\Lambda_{\xi_j}} \leq \Upsilon_j$, $\Lambda_{\xi_j} > 0, \quad j = 1,2$ (these matrices "normalize" the components to make possible to work with values of different physical nature). Besides, $\xi_{1,t}$ and $\xi_{2,t}$ do not violate the existence of the solution to ODE (14).

E) $A^{(0)}$ is Hurwitz and the pair $(A^{(0)}, C)$ is observable.

F) $\tilde{f}_t$ is bounded, specifically, $\|\tilde{f}_t\|^2_{\Lambda_f} \leq \tilde{f}_0 + \tilde{f}_1\|x_t\|^2_{\Lambda_{\tilde{f}}}$, $\Lambda_f > 0, \quad \Lambda_{\tilde{f}} > 0$.

It is worth mentioning that the preceding assumptions are generally met for physically meaningful dynamic systems and a nuclear reactor is not exception.

4.2. Observer structure

Let us define DNN observer as follows:

$$\frac{d}{dt}\hat{x}_t = A^{(0)}\hat{x}_t + W_t\sigma(\hat{x}_t) + B^{(0)}u_t + K_1[y_t - C\hat{x}_t] + K_2\mathbf{sign}(y_t - C\hat{x}_t) \qquad (17)$$

where $\hat{x}_t \in \Re^n$ is the estimated state, $W_t \in \Re^{m\times n}$ is the weight matrix and

$$sign(z) := \begin{cases} 1 \text{ if } z > 0 \\ -1 \text{ if } z < 0 \\ \in [-1,1] \text{ if } z = 0 \end{cases} \qquad (18)$$

One can see that the structure of the observer (17) consists of three parts (see fig. 1):

- the neural network identifier with a single output layer

$$A^{(0)}\hat{x}_t + W_t\sigma(\hat{x}_t) + B^{(0)}u_t$$

- the Luenberger tuning term $K_1[y_t - C_t\hat{x}_t]$;
- the sign correction term $K_2\mathbf{sign}(y_t - C\hat{x}_t)$ which is intended to reduce the output external noise effect associated with real data. In general, this term improve the global performance of the observer particularly when the output error

$[y_t - C\hat{x}_t]$ is small and consequently the Luenberger term is not effective anymore.

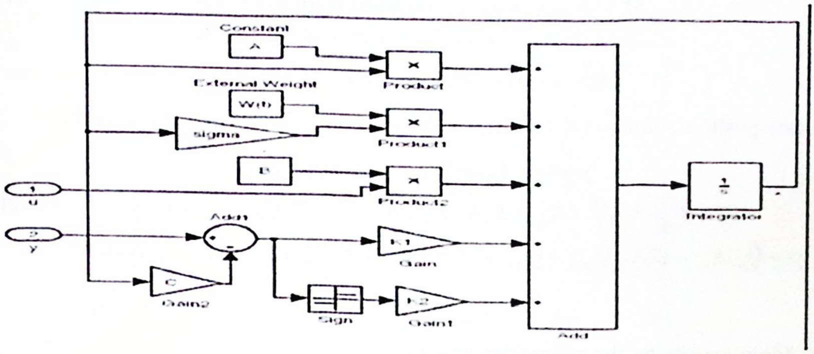

Fig. 1. Structure of DNN observer.

4.3. Off-line training of the network

Before using DNN observer (17) on-line, it is necessary to select adequate values for $A^{(0)}$, $B^{(0)}$, and $W^{(0)}$. This preliminary process of selection is known as off-line training of DNN. If the off-line knowledge of all states of (14) is not available then at least it is necessary to resort to a simplified and inclusive imprecise model of (14) as the generator of N training data $(u_{t_k}, x_{t_k})|_{k=1,N}$. Basically, the training process consists of two stages: First, using "try-to-test" method, values for $A^{(0)}$ and $B^{(0)}$ are proposed such that assumption E is satisfied. Next, using a least square method, the best nominal value of $W^{(0)}$ is determined. Let us define this value as $W^{(0)*}$ = $\min \sum_{k=1}^{N} \| \mathcal{J}_{t_k} \|^2 = [Y_{t_1} \ \cdots \ Y_{t_N}][\sigma(x_{t_1}) \ \cdots \ \sigma(x_{t_N})]^+$ where $Y_{t_k} := \dot{x}_{t_k} - A^{(0)}x_{t_k} - B^{(0)}u_{t_k}$ (here $[\cdot]^+$ means pseudoinverse in Moore-Penrose sense). Starting from t_N, we are ready to initiate the learning (or DNN adaptation) process.

4.4. Learning law

The weight matrix W_t is adjusted by the the following learning law:

$$
\begin{aligned}
&\dot{W}_t^{(i,j)} = -k\mu_t S_t^{(i,j)} \mathbf{sign}\left(\tilde{W}_t^{(i,j)}\right),\ i,j=\overline{1,n} \\
&\mu_t := \left\| N_\delta P \tilde{W}_t^T \sigma(\hat{x}_t) \right\|_\Pi^2 + 2e_t^T C N_\delta P \tilde{W}_t \sigma(\hat{x}_t) \\
&S_t^{(i,j)} = 1/n,\ i,j=\overline{1,n}\ \text{(uniform learning)},\ k>0 \\
&\Pi := C^T \Lambda_{\xi_2} C + \delta\Lambda_1,\ \tilde{W}_t := W_t - W^{(0)*},\ \delta>0 \\
&e(t) := y(t) - C\hat{x}(t),\ N_\delta := \left(C^T C + \delta I\right)^{-1}
\end{aligned}
\tag{19}
$$

P is the positive solution (if it exists) for the algebraic Riccati equation given by

$$
\begin{aligned}
&P\tilde{A}^{(0)*} + \left(\tilde{A}^{(0)*}\right)^T P + PRP + Q = 0 \\
&R = W_{\Lambda_\sigma^{-1}} + \Lambda_{\tilde{f}}^{-1} + \Lambda_{\xi_1}^{-1} + K_1 \Lambda_{\xi_2}^{-1} K_1^T,\ \Lambda_{\tilde{f}}\ \tilde{f}_1 \le \eta_1,\ Q_0>0,\ \Lambda_\sigma>0 \\
&Q = \left(l_\sigma \|\Lambda_\sigma\| + \delta\|\Lambda_1\|\right) I_{n\times n} + Q_0,\ \tilde{A}^{(0)*} = \left(A^{(0)} + K_1 C\right),\ W_{\Lambda_\sigma^{-1}} = \left(W^{(0)*}\right)^T \Lambda_\sigma^{-1} W^{(0)*}
\end{aligned}
\tag{20}
$$

4.5. Main results on the estimation process

One of the principles advantages of (17) and of the corresponding learning law (19), it is the possibility of guaranteeing that both averaged estimation error and weights are upperly bounded. In fact,

Theorem 1. *If there exist positive definite matrices* $\Lambda_{\tilde{f}}, \Lambda_{\xi_1}, \Lambda_{\xi_2}, \Lambda_\sigma, \Lambda_1, Q_0$ *and positive constants* δ, k *such that the matrix Riccati equation (20) has a positive definite solution, then the DNN observer (17), supplied by the learning law (19), with any matrix* K_1 *guarantying that* $\tilde{A}^{(0)*}$ *is stable, that is,* $\tilde{A}^{(0)*} := \left(A^{(0)} + K_1 C\right)$ *is Hurwitz and* $K_2 = \lambda P^{-1} C^T$, $\lambda > 0$ *provides the following upper bound for the averaged estimation error*

$$
\begin{aligned}
&\overline{\lim_{T\to\infty}} \frac{1}{T} \int_{t=0}^{T} \|\Delta_t\|_P^2 dt \le \overline{\lim_{T\to\infty}} \frac{1}{T} \int_{t=0}^{T} \left(\|\Delta_t\|_P^2 + 2\lambda \sum_{i=1}^{n} \left| C\Delta(t)_i \right| \right) dt \le \frac{\rho_Q}{\alpha_Q} \\
&\rho_Q := \tilde{f}_0 + \Upsilon_1 + 2\Upsilon_2 + 4\lambda\sqrt{n\Upsilon_2},\ \alpha_Q := \lambda_{\min}\left(P_1^{-1/2} Q P_1^{-1/2}\right) > 0
\end{aligned}
\tag{21}
$$

Lemma 1. *For the learning law (19) and for any* $\rho \in \left(0, \lambda_{\min}(\Pi)\right)$ *the following property holds:*

$$
\overline{\lim_{T\to\infty}} \frac{1}{T} \int_{t=0}^{T} \left\| \tilde{W}_t \sigma(\hat{x}_t) \right\|^2 dt \le \frac{\overline{\lim_{T\to\infty}} \frac{1}{T} \int_{t=0}^{T} \left\| C^T e_t \right\|_{(\Pi-\rho I)^{-1}}^2 dt}{\rho \lambda_{\min}\left(P N_\delta^2 P\right)}
\tag{22}
$$

The proofs of theorem 1 and lemma 1 are achieved by means of Lyapunov-like analysis in [17].

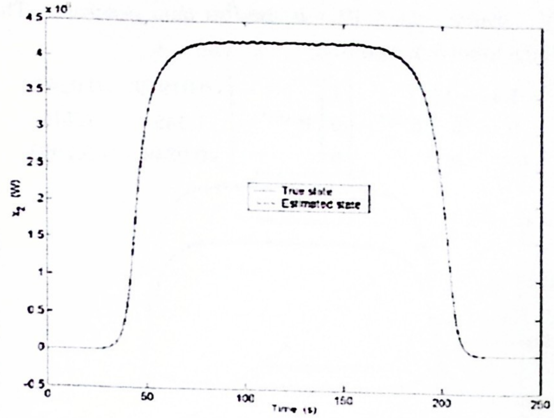

Fig. 2. The estimation process for the state $x_{2,t}$.

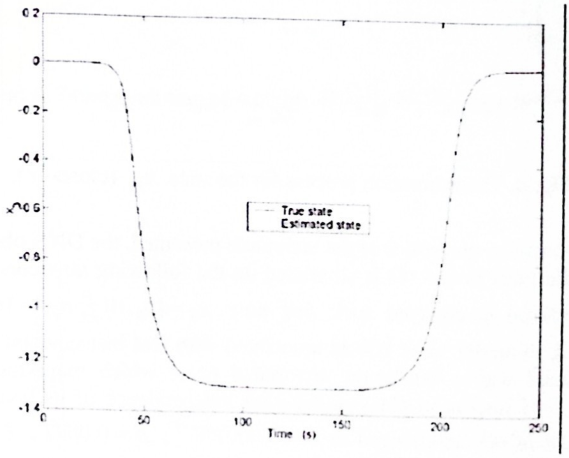

Fig. 3. The estimation process for the state $x_{3,t}$.

5. Numerical Example

In this example the state estimation process of a nuclear reactor via the DNN observer (17) is illustrated. In first term, to overcome the measuring complexity or else the absence of adequate sensors that prevent the off-line knowledge of all states, *the model* (8) *with nominal parameters given in the section 2 and with nominal initial condition*

$x_0 = [n_0, \frac{\beta}{\lambda\Lambda} n_0, 0]^T$ *where* $n_0 = 1W$ *is used as data generator.* The preliminary training produces the following results:

$$A^{(0)} = \begin{bmatrix} -1.2 & -1.3 & -1 \\ 1 & 0 & 0 \\ 0 & 0.1 & 0 \end{bmatrix}, B^{(0)*} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, W^{(0)*} = \begin{bmatrix} -0.01919 & -0.02472 & 0.1183 \\ 1.345 & 1.744 & 0.2911 \\ -0.0844 & -0.1093 & -0.02583 \end{bmatrix}$$

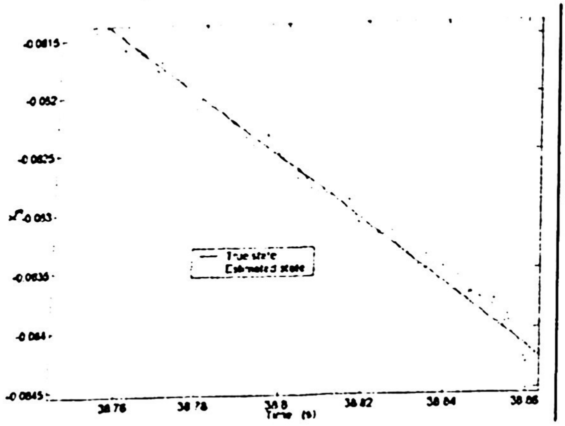

Fig. 4. The estimation process for the state $x_{3,t}$ (close-up).

To show the robustness properties of the technique presented, the DNN observer (17) is proved when the plant model (8) is simulated on the following new conditions: First, the initial condition is changed such that now $x_0' = [n_0, 10 \frac{\beta}{\lambda\Lambda} n_0, 1\times 10^{-6}]^T$ where $n_0 = 1W$. Second, to model some effects associated with real instrumentation, the plant output is polluted with a uniformly distributed noise which magnitude is always approximately 3 % respect to plant output with independence of its level. Third, the parameter values of (8) are changed to $\alpha = 0.0097°C^{-1}$, $\beta = 0.0072$, $\lambda = 0.3942s^{-1}$, and $\Lambda = 30\mu s$ (both γ and K stay equal). Although, apparently these changes are small, as it will be seen a Luenberger observer basis on (8) with nominal values of the section 2 is not able to work satisfactorily on these new conditions (of course, these changes are supposed to be unknown for the DNN observer). By "try-to-test" method, the parameters of DNN observer (17) are selected as follows:

$K_1 = [-2.7, 15, -0.1]^T$, $K_2 = [0.054, -0.037, 0.079]^T$, $k = 1.2$ and

$$P = 10\begin{bmatrix} 9 & 4 & -3 \\ 4 & 10 & 2 \\ -3 & 2 & 3 \end{bmatrix}.$$

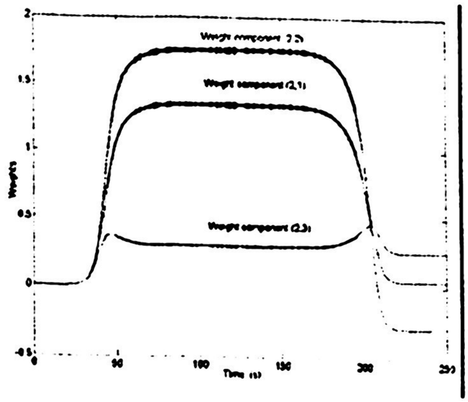

Fig. 5. Time evolution of weights $W_t^{(2.1)}$, $W_t^{(2.2)}$ and $W_t^{(2.3)}$.

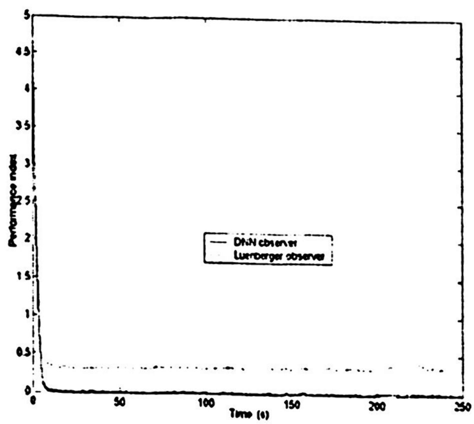

Fig. 6. Performance index for global estimation process.

The results of estimation process are displayed in Figs. 2-6. In Fig. 2 the estimated state masks almost completely to the true state $x_{2,t}$. In Fig. 3 the difference between true state $x_{3,t}$ and estimated state $\hat{x}_{3,t}$ is practically imperceptible. However, a closer look provided by Fig. 4 illustrates better such difference. Some elements of weight matrix are presented in Fig. 5. It is important to mention that due to wide range of state values associated with (8) it is necessary to resort to the normalization of variables in both the preliminary training and on-line estimation process. Such normalization does not affect the estimation results. Instead, it permits to the observer (17) works satisfactorily. To quantify the global performance of an observer, let us define the following performance index:

$$J_t = \frac{1}{t+\varepsilon}\int_{s=0} \left\| x_s - \hat{x}_s \right\|_{Q_0}^{2} ds, Q_0 > 0, \varepsilon = 0.01. \tag{23}$$

Performance indexes of the observer studied here and a Luenberger observer are shown in Fig. 6 for comparison.

6. Conclusions

We have studied the use of DNN observers with SM learning law for nuclear reactor state estimation in the presence of measurement noise and uncertainty. As it was seen in the numerical example, this technique represents a significant advantage respect to a simple Luenberger observer. Since it permits the possibility of utilizing reduced order systems besides guaranteeing the averaged boundness of both the DNN weights and the estimation error, we conclude that this methodology is promising in nuclear reactor applications and may constitute the basis for posterior development of efficient feedback controller designs.

References

[1] H. Arab-Alibeik and S. Setayeshi, Improved temperature control of a PWR nuclear reactor using an LQG/LTR based controller, *IEEE Trans. Nucl. Sci.*, vol. 50, no. 1, pp. 211-218, Feb 2003.

[2] Q. Li and J. A. Bernard, Design and evaluation of an observer for nuclear reactor fault detection, *IEEE Trans. Nucl. Sci.*, vol. 49, issue 3, part 2, pp. 1304-1308, Jun 2002.

[3] A. Tornambè, Use of asymptotic observers having high-gains in the state and parameter estimation, in Proc. of the 28th Conf. on Decision and Control, Tampa, Florida, 1989, pp. 1791-1794.

[4] S. Haykin, *Neural Networks: A Comprehensive Foundation*, IEEE Press, NewYork, 1994.

[5] N. S. Garis, I. Pázsit, U. Sandberg, and T. Andersson, Determination of PWR control rod position by core physics and neural network methods, *Nucl. Technol.*, vol. 123, pp. 278-295, Sep. 1998.

[6] K. Nabeshima, E. Ayaz, S. Seker, B. Barutcu, E. Turkcan, and K. Kudo, Nuclear Power Plant Monitoring with MLP and RBF Network, *Proc. of the 6th International FLINS Conference on Applied Computational Intelligence*,

Blankenberge, Belgium, September, 2004.
[7] M.G. Na, S.H. Shin, D.W Jung, S.P Kim, J.H. Jeong, and B.C. Lee. Estimation of Break Location and Size for Loss of Coolant Accidents Using Neural Networks, *Proc. of the 6th International FLINS Conference on Applied Computational Intelligence*, Blankenberge, Belgium, September. 2004.
[8] M. N. Khajavi, M. B. Menhaj, and A. A. Suratgar, A neural network controller for load following operation of nuclear reactors, in Proc. Int. Joint Conf. on Neural Networks, Washington, DC, July 2001, pp. 491-496.
[9] M. Boroushaki, M. B. Ghofrani, C. Lucas, and M. J. Yazdanpanah Identification and control of a nuclear reactor core (VVER) using recurrent neural networks and fuzzy systems, *IEEE Trans. Nucl. Sci.*, vol. 50, no. 1, pp. 159-174, Feb. 2003.
[10] V. I. Utkin, "*Sliding Modes in Control Optimization*," Springer-Verlag, Berlin, 1992.
[11] P. Wang, T. Aldemir, and V. I. Utkin, Estimation of xenon concentration and reactivity in nuclear reactors using sliding mode, in Proc. of the 40th Conf. on Decision and Control, Orlando, Florida, Dec. 2001, pp. 1801-1806.
[12] Y. B. Shtessel, Sliding mode control of the space nuclear reactor system, *IEEE Trans. Aerosp. Electron. Syst.*, vol. 34, no. 2, pp. 579-589, Apr. 1998.
[13] I. Chairez, A. Poznyak and T. Poznyak, "Model Free Sliding Mode Neural Observer for Ozonation Reaction", *Proc. of the 8-th Int. Workshop on Variable Structure Systems*, Villanova i la Geltrú, Spain, 6-8 September, 2004
[14] D. L. Hetrick, Dynamics of Nuclear Reactors, Editorial The University of Chicago Press. 1971.
[15] J. Viais, Cálculo de los Parámetros Fundamentales para el Estudio Dinámico del Reactor TRIGA Mark III del Centro Nuclear de México, B. Sc. Thesis, Universidad Nacional Autónoma de México, 1994.
[16] A. Sabanovic, Deterministic output noise effects in sliding mode observation, in Variable Structure Systems: From Principles to Implementation, Institution of Electrical Engineers, 2004, ch. 3..
[17] I. Chairez, A. Poznyak and T. Poznyak, New Sliding Mode Learning Law for Dynamic Neural Network Observer, *IEEE Trans. Circuits Syst. II, to appear.*